

Chapter 1

$$i(t) \&= \frac{dq}{dt} \Leftrightarrow q(t) = \int i(t) \cdot dt \quad P = v \cdot i = i \cdot \frac{W}{q} \Leftrightarrow v = \frac{W}{q} \Leftrightarrow W = v \cdot q = \int v \cdot i \cdot dt$$

Chapter 4

$$\begin{aligned} \text{Load line: } i_x &= -\frac{v_x}{R_T} + \frac{v_t}{R_T} \quad \text{General: } A_v &= \frac{v_{out}}{v_{in}} \\ \text{Inverting: } A_v &= -\frac{R_f}{R_{in}} \quad \text{Non Inverting: } A_v &= 1 + \frac{R_f}{R_{in}} \end{aligned}$$

Inverting

Non-inverting

Chapter 5

Capacitor

$$C \&= \frac{q}{v} \quad i(t) = C \frac{dv}{dt} \Leftrightarrow v(t) = \frac{1}{C} \int_0^t i(t) \cdot dt$$

$$\begin{aligned} \text{Series: } \frac{1}{C_T} &= \sum_{i=0}^N \frac{1}{C_i} \quad \text{Parallel: } C_T &= \sum_{i=0}^N C_i \\ \text{Energy: } E &= \frac{1}{2} C v^2 \end{aligned}$$

Differential equation solution

Where $v_s = v_{\infty}$:

$$\tau = R \cdot C$$

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\tau &= R \times C \\

v_C(t) &=
\begin{cases}
\begin{array}{lr}
v_0 & & t \leq 0 \\
v_{\infty} + (v_0 - v_{\infty}) e^{-t/\tau} & & t > 0
\end{array} \\
\end{cases} \\

& v_0 e^{-t/\tau} \text{ (Natural response, no input)} \\
& v_{\infty} \left(1 - e^{-t/\tau}\right) \text{ (Forced response, input)} \\

i_C(t) &= \frac{v_s - v_C(t)}{R} = \frac{-(v_0 - v_{\infty}) e^{-t/\tau}}{R}
    
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$$\end{aligned}$$

1. Remove all independent sources, find equivalent resistance and capacitance, find τ .
2. Set C as open circuit, find initial capacitor voltage v_0 at $t=0$
3. Set C as open circuit, find final capacitor voltage v_∞ at $t \rightarrow \infty$

Inductor

$$L \frac{di}{dt} \rightarrow i(t) = \frac{1}{L} \int_0^t v dt$$

$$i_T = \sum_{i=0}^N i_i$$

$$E = \frac{1}{2} Li^2$$

Differential equation solution

Where $v_s/R = i_\infty$:

$$\tau = \frac{L}{R}$$

$$i(t) = \begin{cases} i_0 & t \leq 0 \\ i_\infty + (i_0 - i_\infty)e^{-t/\tau} & t > 0 \end{cases}$$

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\tau = \frac{L}{R}

i(t) =
\begin{cases}
\begin{array}{lr}
i_0 & t \leq 0 \\
i_\infty + (i_0 - i_\infty)e^{-t/\tau} & t > 0
\end{array}
\end{cases}

& i_0 e^{-t/\tau} \text{ (Natural response, no input)} \\
& i_\infty (1 - e^{-t/\tau}) \text{ (Forced response, input)}

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$$\tau = \frac{L}{R}$$

Voltage drop in DC for capacitor and inductor at steady state

<p>CAPACITOR:</p> <pre> v_T _ <- V_1 C1 =) <- V_D1 C2 = ... CN = GND *</pre>	<p>INDUCTOR:</p> <pre> v_T _ <- V_1 L1 3) <- V_D1 C2 3 ... LN 3 GND *</pre>
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Capacitor

Current through capacitors in series is the same, so all capacitors have same charge stored \$q\$.

$$\begin{aligned} \text{Voltage drop over capacitor } i: & \quad v_{Di} \quad \&= \quad v_T \frac{C_T}{C_i} \\ \text{Voltage divider: } & \quad v_i \\ \&= \quad v_T \frac{C_T}{\frac{1}{C_i} + \frac{1}{C_{i+1}} + \dots + \frac{1}{C_N}} \end{aligned}$$

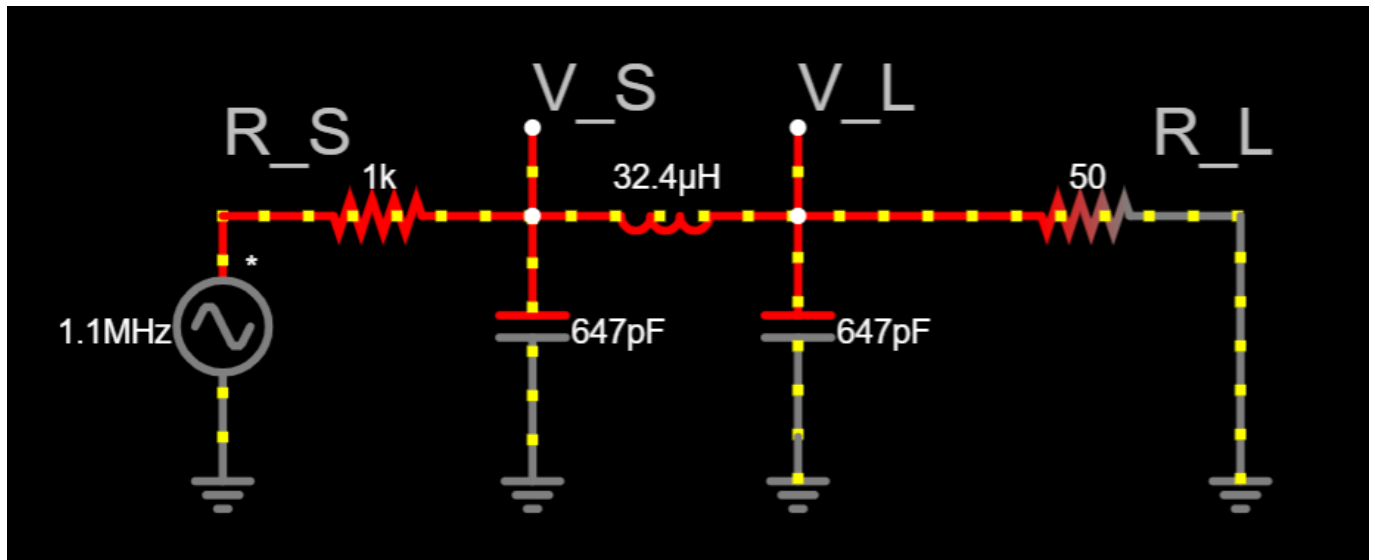
Inductor

No voltage drop in steady state (Inductor is a short circuit)

Chapter 7

Maximum power transfer in AC

$$\begin{aligned} \text{Condition: } & \quad \overline{Z_L} = \overline{Z_S^*} \\ \text{Maximum power to load (50%): } & \quad 2P_{\text{avg}} = P_{\text{rms}} \cdot \sqrt{2} = P_{\text{max}} = \frac{|V_S|^2}{4R_S} \quad \&= \quad \frac{|V_L|^2}{4R_L} \\ \text{Total maximum power: } & \quad 2P_{\text{avg}} = P_{\text{rms}} \cdot \sqrt{2} = P_{\text{max}} = \frac{|V_S|^2}{2R_S} \end{aligned}$$



Complex Power

Where $\bar{V} = V \angle \theta$ and $\bar{I} = I \angle \phi$:

$$\begin{aligned} \text{Complex [VA]: } & \quad \bar{S} \quad \&= \quad \bar{V} \text{rms} \times \bar{I} \text{rms}^* = \\ & \quad \frac{\bar{V} \times \bar{I}^*}{2} = \frac{VI}{2} \angle (\theta - \phi) \end{aligned}$$

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\text{Apparent [VA]: } |S| \\
\text{Real [W]: } P \quad \&= \quad |S| \cos(\theta - \phi) = \text{Re}(\bar{S}) \\
\text{Reactive [VAR]: } Q \quad \&= \quad |S| \sin(\theta - \phi) = \text{Im}(\bar{S}) \\
Q \quad \&= \quad P \tan(\arccos(\text{Power factor})) \\
\text{Power Factor} \quad \&= \quad \frac{P}{|S|} = \cos\left(\arctan\left(\frac{Q}{P}\right)\right)
    
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\end{align} \$\$\$

Where $\bar{S} = |S| \angle \varphi$

$$\varphi = \arctan\left(\frac{Q}{P}\right)$$

	Lagging	Leading
Voltage	Current behind	Current ahead
Load type	Inductive	Capacitive
Q	$Q > 0$	$Q < 0$
φ	$\varphi > 0$	$\varphi < 0$

Chapter 8

Constants

$$\mu_0 = 4\pi \times 10^{-7}$$

$$\begin{aligned} \text{Faraday's law: } \epsilon &= -N \frac{d\varphi}{dt} \quad \text{Ampere's law: } B &= \\ \frac{\mu_0 I}{2\pi r} \end{aligned}$$

Transformer

Step up: $n > 1$

Step down: $n < 1$

$$\begin{aligned} \frac{V_s}{V_p} &= \frac{N_s}{N_p} = \frac{i_p}{i_s} = n \quad \bar{Z}_{in} = \frac{1}{n^2} \bar{Z}_L \\ \end{aligned}$$

Motor

For permanent motors, define permanent torque constant $k_{TP} = k_T \varphi$

$$\begin{aligned} T &= k_T \varphi i_a = k_{TP} i_a \quad P_{mech} = \omega_{mech} T \\ &= \omega_{mech} k_{TP} i_a \end{aligned}$$

Back emf

Define for permanent motors, define permanent armature constant $k_{aP} = k_a \varphi$

Note, back emf should oppose v_a and i_a

$$\begin{aligned} e_b &= k_a \varphi \omega_{mech} = k_{aP} \omega_{mech} \\ \end{aligned}$$

Summary

For ideal motor, torque and armature constants are the same: $k_a = k_T$

Define p as number of magnetic poles and M as the number of parallel paths in armature winding.

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$$P_e = e_b \times i_a = k_{aP} \times \omega_{\text{mech}} \times i_a$$


$$k_a = k_T = \frac{pN}{2\pi M}$$


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For permanent magnet DC motor in DC steady state:

Define viscous frictional damping coefficient b and load torque T_L

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$$\begin{cases} 0 = v_a - i_a R_a - k_{aP} \omega_{\text{mech}} = v_a - i_a R_a - e_b \\ k_{TP} i_a = T_L + b \omega_{\text{mech}} \end{cases}$$


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$$\begin{cases} 0 = v_a - i_a R_a - k_{aP} \omega_{\text{mech}} = v_a - i_a R_a - e_b \\ k_{TP} i_a = T_L + b \omega_{\text{mech}} \end{cases}$$

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\end{cases}

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&\text{Analog speed control (Voltage): } T = \frac{k_{TP}}{R} v_s - \frac{k_{TP} k_{aP}}{R} \omega_{\text{mech}}

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&\text{Analog speed control (Current): } T = \frac{k_{TP} R_S}{R} i_s - \frac{k_{TP} k_{aP}}{R} \omega_{\text{mech}}

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\end{align}

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